$$x_1 = y_1 - y_1(1-y_1)\left[\frac{\sum_{i=1}^{N} x_i z_i}{P} - \frac{v_l}{RT}\right]\left(\frac{\partial P}{\partial y_1}\right)_{T,n_j} \quad (15)$$

This can be iterated by assuming the vapor phase to be a perfect gas on the first trial. In this event, $\frac{\sum_i x_i z_i}{P} = \frac{1}{P}$, which is identical to Equation (10) with the last term neglected.

NOTATION

f = fugacity
n = number of moles
N = number of components in a solution
P = total pressure
p = partial pressure
$\overline{V}$ = partial molar volume
v = molar volume
x = liquid-phase mole fraction
y = gas-phase mole fraction
z = compressibility factor
ϕ = fugacity coefficient, f/p

Subscripts

i = any component or all components
j = all components but one
k = any component
l = liquid phase
T = total
1 = component 1

Superscripts

0 = pure component

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Friction Factors and Pressure Drops for Sinusoidal Laminar Flow in an Annulus

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Hershey and Song (*1*) examined the friction factors and pressure drops for sinusoidal flow of water and blood in rigid tubes. As pointed out in their paper, the pulsatile flow phenomenon is of great importance in many engineering applications, such as in pumping systems and heat and mass transfer operations. This analysis is concerned with the same problem in an annulus. The problem is first solved for the generalized case, where the initial velocity distribution is an arbitrary function of radial coordinate and the pressure gradient an arbitrary time-dependent quantity. The governing momentum equation is solved by double transformation technique. The solution agrees with that of special case (*2*).

The Navier-Stokes equation reduces, under the present assumptions, to

$$\frac{\partial u}{\partial t} = -\frac{g_c}{\rho}\frac{dp}{dz}(t) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \quad (1)$$

and the appropriate boundary conditions are

$$u(r, 0) = U(r) \quad (2)$$

$$u(R_1, t) = 0 \quad (3)$$

$$u(R_2, t) = 0 \quad (4)$$

To solve Equation (1) subject to conditions (2) to (4), we first use the finite Hankel transformation (3) with respect to r, defined by

$$\overline{u}(\xi_i, t) = \int_{R_1}^{R_2} r u(r, t) Z_0(\xi_i r)\, dr \quad (5)$$

where

$$Z_0(\xi_i r) = J_0(\xi_i r) Y_0(\xi_i R_2) - J_0(\xi_i R_2) Y_0(\xi_i r) \quad (6)$$

and the eigenvalues ξ_i are the positive roots of

$$J_0(\xi R_1) Y_0(\xi R_2) - J_0(\xi R_2) Y_0(\xi R_1) = 0 \quad (7)$$

Then, the Laplace transformation with respect to t is defined by

$$\overline{\overline{u}}(\xi_i, s) = \int_0^\infty \overline{u}(\xi_i, t) e^{-st} dt \quad (8)$$

Equation (1) with the conditions (2) to (4) is transformed, with certain arrangement, to

$$\overline{\overline{u}}(\xi_i, s) = \frac{\overline{U}(\xi_i)}{s + \nu\xi_i^2} - \frac{1}{s + \nu\xi_i^2}\frac{dp}{dz}(s)$$

$$\frac{2}{\pi\xi_i^2}\,\frac{J_0(\xi_i R_1)-J_0(\xi_i R_2)}{J_0(\xi_i R_1)} \tag{9}$$

where

$$\overline{U}(\xi_i)=\int_{R_1}^{R_2} rU(r)Z_0(\xi_i r)\,dr \tag{10}$$

By inverting the foregoing transformed Equation (9) and by using the convolution integral, the solution takes the form

$$u(r,t)=\frac{\pi^2}{2}\sum_{i=1}^{\infty}\frac{\xi_i^2 J_0^2(\xi_i R_1)Z_0(\xi_i r)}{J_0^2(\xi_i R_1)-J_0^2(\xi_i R_2)}\int_{R_1}^{R_2} rU(r)Z_0(\xi_i r)\,dr-\frac{\pi g_c}{\rho}\sum_{i=1}^{\infty}\frac{J_0(\xi_i R_1)Z_0(\xi_i r)}{J_0(\xi_i R_1)+J_0(\xi_i R_2)}\int_0^t\frac{dp}{dz}(\tau)\,e^{-\nu\xi_i^2(t-\tau)}\,d\tau \tag{11}$$

Consider the pulsatile flow where the fluid is initially kept at rest and starts to move by applying a sinusoidal pressure. Following Hershey and Song, the down stream pressure is represented by a modified Fanning equation:

$$p=\left(p_0-\frac{\rho f\overline{V}^2 z}{R_h g_c}\right)(1+\sin\omega t) \tag{12}$$

where $p_0(1+\sin\omega t)$ is the upstream pressure applied. Hence we have

$$-\frac{dp}{dz}(t)=\frac{\rho f\overline{V}^2}{R_h g_c}(1+\sin\omega t) \tag{13}$$

By inserting Equation (13) and $U(r)=0$ and by noting that by Fourier-Bessel expansion we get

$$\pi\sum_{i=1}^{\infty}\frac{J_0(\xi_i R_1)Z_0(\xi_i r)}{\xi_i^2[J_0(\xi_i R_1)+J_0(\xi_i R_2)]}=\frac{1}{4}\left[R_2^2-r^2+\frac{R_1^2-R_2^2}{\mathrm{Ln}\left(\frac{R_1}{R_2}\right)}\mathrm{Ln}\left(\frac{r}{R_2}\right)\right] \tag{14}$$

Equation (11) reduces to

$$u(r,t)=\frac{\rho f\overline{V}^2}{\mu R_h}\left\{\frac{1}{4}\left[R_2^2-r^2+\frac{R_1^2-R_2^2}{\mathrm{Ln}\left(\frac{R_1}{R_2}\right)}\mathrm{Ln}\left(\frac{r}{R_2}\right)\right]+\pi\sum_{i=1}^{\infty}\frac{J_0(\xi_i R_1)Z_0(\xi_i r)}{\xi_i^2 J_0(\xi_i R_1)+J_0(\xi_i R_2)}\left[\left(\frac{\omega\nu\xi_i^2}{\nu^2\xi_i^4+\omega^2}+1\right)e^{-\nu\xi_i^2 t}+\frac{\nu\xi_i^2[\nu\xi_i^2\sin(\omega t)-\omega\cos(\omega t)]}{\nu^2\xi_i^4+\omega^2}\right]\right\} \tag{15}$$

By the definition of the average velocity, we have

$$\overline{V}=\frac{\int_0^{2\pi}\int_{R_1}^{R_2}u(r,t)\,r\,dr\,dt}{(R_2^2-R_1^2)} \tag{16}$$

Substitution of Equation (15) into (16) yields

$$f=\frac{\pi^2[(R_2^2-R_1^2)(R_2-R_1)]}{2Q}$$

$$\times\frac{1}{\frac{\pi(R_2^2-R_1^2)}{8}\left[R_1^2+R_2^2+\frac{R_2^2-R_1^2}{\mathrm{Ln}\left(\frac{R_1}{R_2}\right)}\right]-2\sum_{i=1}^{\infty}\left[\left(\frac{\omega}{\nu^2\xi_i^4+\omega^2}-\frac{1}{\nu\xi_i^2}\right)\left(\frac{1-e^{-2\nu\xi_i^2\pi}}{\xi_i^4}\right)\frac{J_0(\xi_i R_1)-J_0(\xi_i R_2)}{J_0(\xi_i R_1)+J_0(\xi_i R_2)}\right]}$$

This relation gives the friction factor as a function of the frequency ω.

A special case of practical interest is the nonsinusoidal flow. Setting $\omega=0$ and combining Equations (15) and (13), we get

$$u(r,t)=\left(-\frac{g_c}{\mu}\frac{dp}{dz}\right)\left\{\frac{1}{4}\left[R_2^2-r^2+\frac{R_1^2-R_2^2}{\mathrm{Ln}\left(\frac{R_1}{R_2}\right)}\mathrm{Ln}\left(\frac{r}{R_2}\right)\right]-\pi\sum_{i=1}^{\infty}\frac{J_0(\xi_i R_1)Z_0(\xi_i r)e^{-\nu\xi_i^2 t}}{\xi_i^2[J_0(\xi_i R_1)+J_0(\xi_i R_2)]}\right\} \tag{18}$$

This solution agrees with that earlier obtained by Müller (2).

NOTATION

f = Fanning friction factor
g_c = Newton's law conversion factor
J_0 = Bessel function of the first kind of zero order
p = pressure
Q = volumetric flow rate
r = radial position
R_1 = outer radius of inner cylinder
R_2 = inner radius of outer cylinder
R_h = hydraulic radius
t = time
u = velocity
U = initial velocity
$\overline{V}$ = average velocity
Y_0 = Bessel function of the second kind of zero order
z = axial position
Z_0 = combination of Bessel function, as defined by Equation (6)

Greek Letters

μ = viscosity of fluid
ν = kinematic viscosity of fluid
ξ = eigenvalues
ρ = density of fluid
ω = frequency

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